C FUNCTION NOTATION

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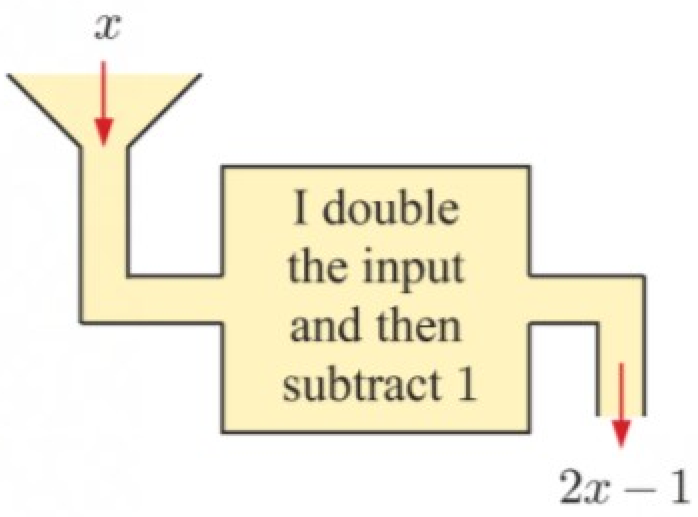
We sometimes use a ‘function machine’ to illustrate how functions behave.

For example, the machine alongside has been programmed to perform a particular function. Whatever number is fed into the machine, the machine will double the number and then subtract 1.



If f is used to represent this particular function, we can write:

f is the function that will convert x into $2x-1$.

If 3 is fed into the machine, $2(3)-1=5$ comes out.

This function can be written as: $\boldsymbol{f : x \mapsto 2x - 1}$

$$\underset{\text{function } f}{\boldsymbol{f}}\ \underset{\text{such that}}{\boldsymbol{:}}\ \underset{x}{\boldsymbol{x}}\ \underset{\text{is converted into}}{\boldsymbol{\mapsto}}\ \underset{2x-1}{\boldsymbol{2x-1}}$$

We can also write this function as $f(x)=2x-1$

For any function f, the value of the function when $x = a$ is given by $f(a)$.

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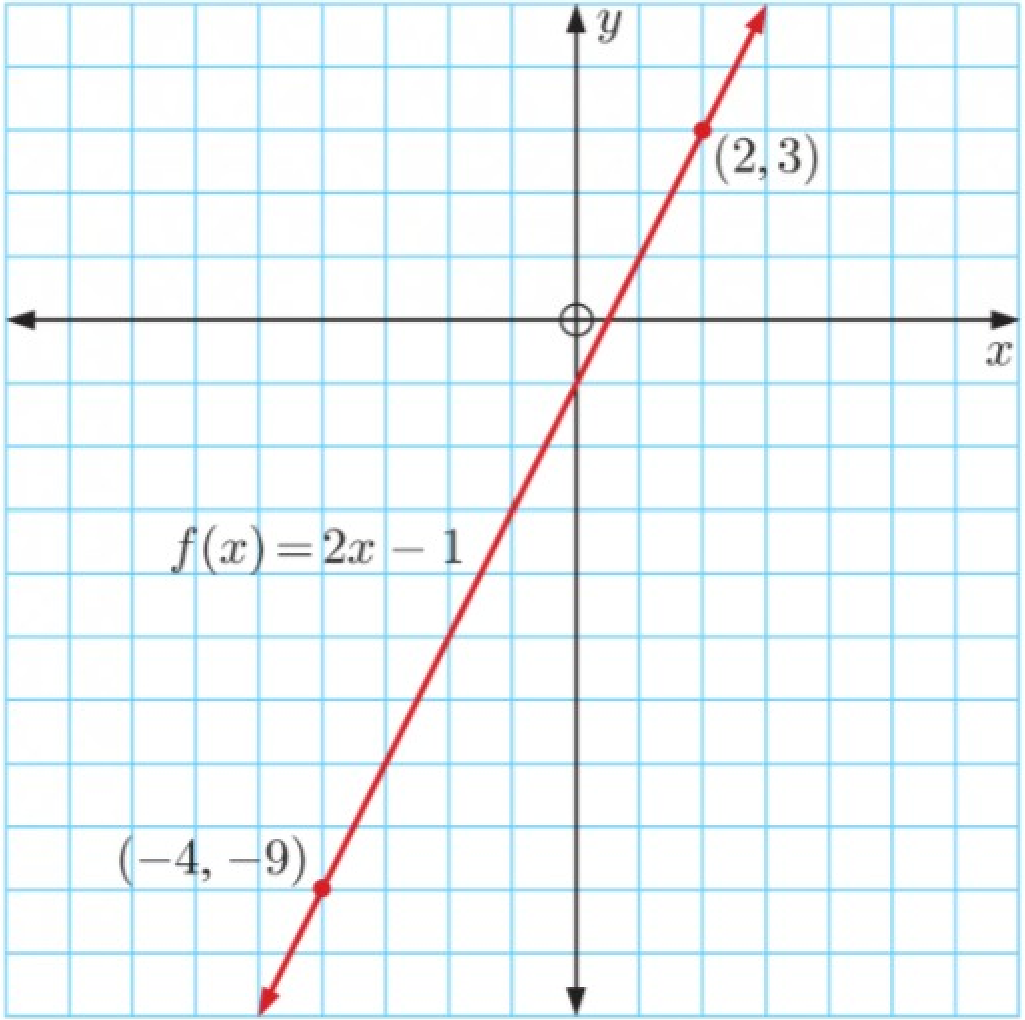

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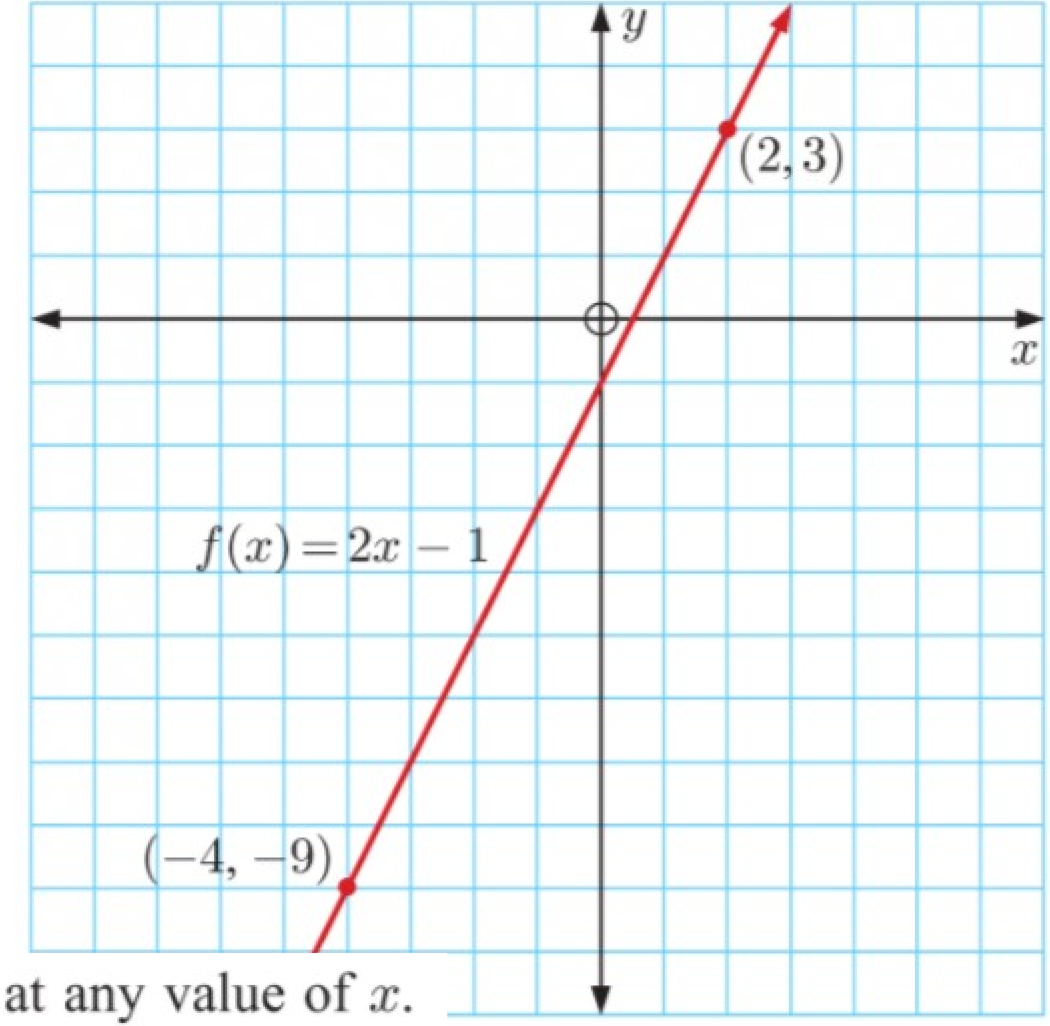


Note that:

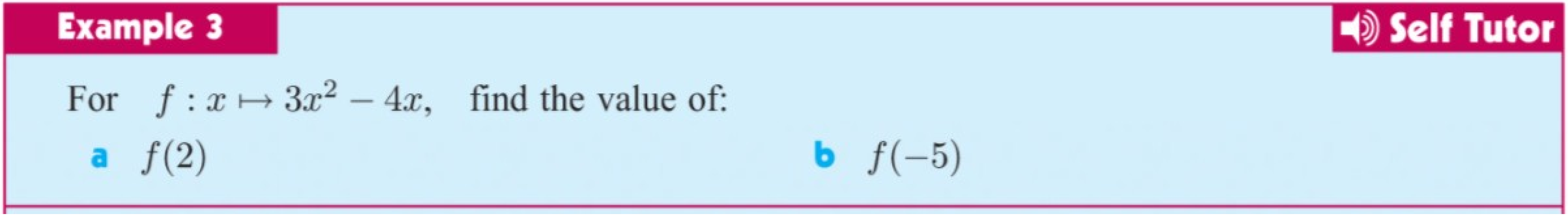
- $f(x)$ is read as "f of x", and is the value of the function at any value of x.
- f is the function which converts x into $f(x)$, so $f : x \mapsto f(x)$.
- $f(x)$ is sometimes called the **image** of x.

Example 3

Self Tutor

For $f : x \mapsto 3x^2 - 4x$, find the value of:

- **a** $f(2)$
- **b** $f(-5)$



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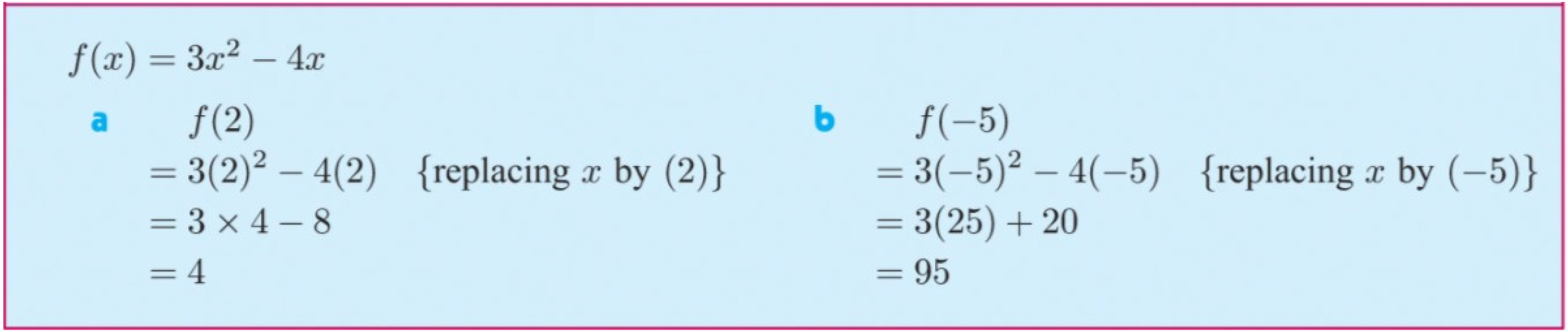


$f(x) = 3x^2 - 4x$

a

$$\begin{aligned} & f(2) \\ &= 3(2)^2 - 4(2) \quad \{\text{replacing } x \text{ by } (2)\} \\ &= 3 \times 4 - 8 \\ &= 4 \end{aligned}$$

b

$$\begin{aligned} & f(-5) \\ &= 3(-5)^2 - 4(-5) \quad \{\text{replacing } x \text{ by } (-5)\} \\ &= 3(25) + 20 \\ &= 95 \end{aligned}$$

Example 4

Self Tutor

For $f(x) = 4 - 3x - x^2$, find in simplest form:

a $f(-x)$

b $f(x+2)$

a
$$\begin{aligned} f(-x) &= 4 - 3(-x) - (-x)^2 && \{\text{replacing } x \text{ by } (-x)\} \\ &= 4 + 3x - x^2 \end{aligned}$$

b
$$\begin{aligned} f(x+2) &= 4 - 3(x+2) - (x+2)^2 && \{\text{replacing } x \text{ by } (x+2)\} \\ &= 4 - 3x - 6 - [x^2 + 4x + 4] \\ &= -x^2 - 7x - 6 \end{aligned}$$

EXERCISE 15C.1

1 For $f : x \mapsto 2x + 3$, find:

a $f(0)$
b $f(2)$
c $f(-1)$
d $f(-5)$
e $f(-\frac{1}{2})$

2 For $g(x) = -5x + 3$, find:

a $g(1)$
b $g(4)$
c $g(-2)$
d $g(-x)$
e $g(x + 4)$

3 For $f(x) = 2x^2 - 3x + 2$, find:

a $f(0)$
b $f(3)$
c $f(-4)$
d $f(-x)$
e $f(x + 1)$

4 For $P : x \mapsto 4x^2 + 4x - 3$, find:

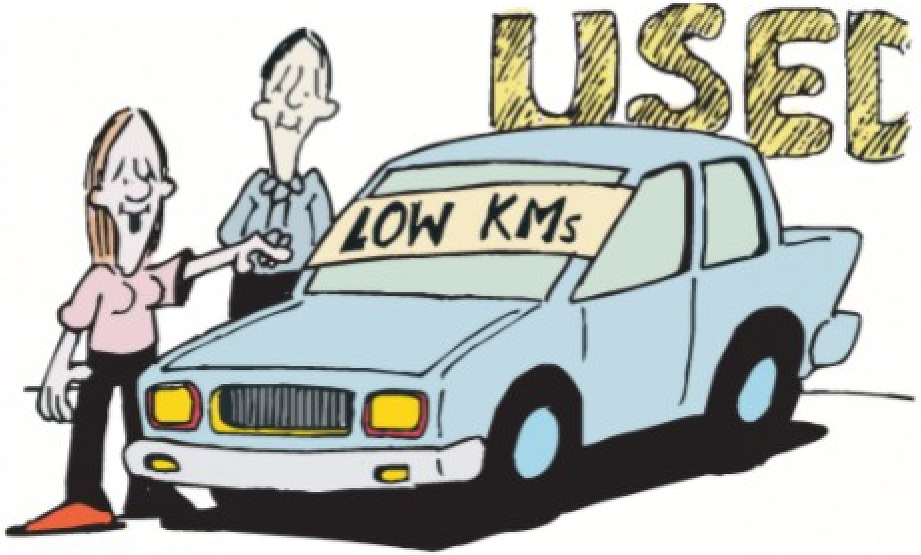
a $P(3)$
b $P(-1)$
c $P(\frac{1}{2})$
d $P(x - 3)$
e $P(2x)$

5 Consider the function $R(x) = \frac{2x-3}{x+2}$.

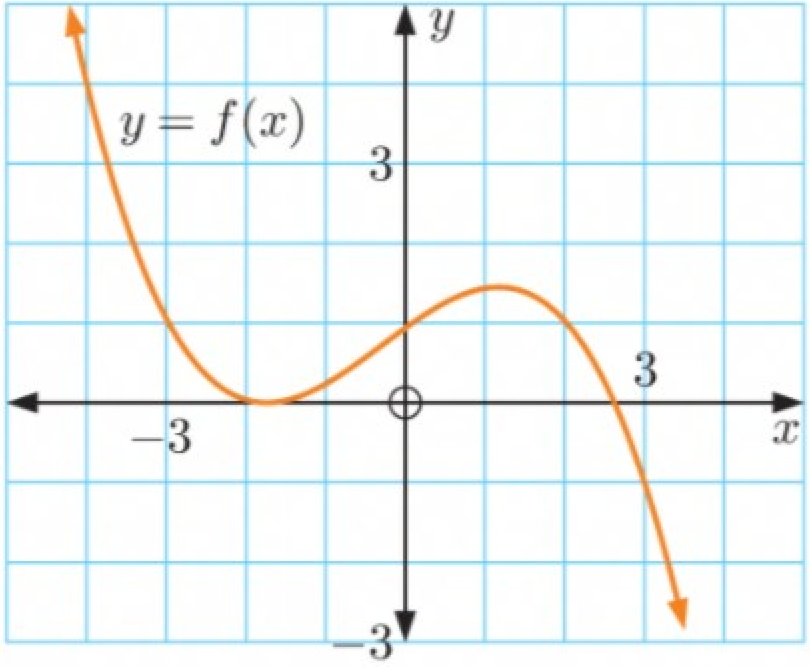
- **a** Evaluate: **i** $R(0)$ **ii** $R(1)$ **iii** $R(-\frac{1}{2})$
- **b** Find a value of x such that $R(x)$ does not exist.
- **c** Find $R(x-2)$ in simplest form.
- **d** Find x such that $R(x) = -5$.

6 The value of a car t years after purchase is given by $V(t) = 28\,000 - 4000t$ dollars.

- **a** Find $V(4)$, and state what this value means.
- **b** Find t when $V(t) = 8000$, and explain what this represents.
- **c** Find the original purchase price of the car.



7 The graph of $y = f(x)$ is shown alongside.

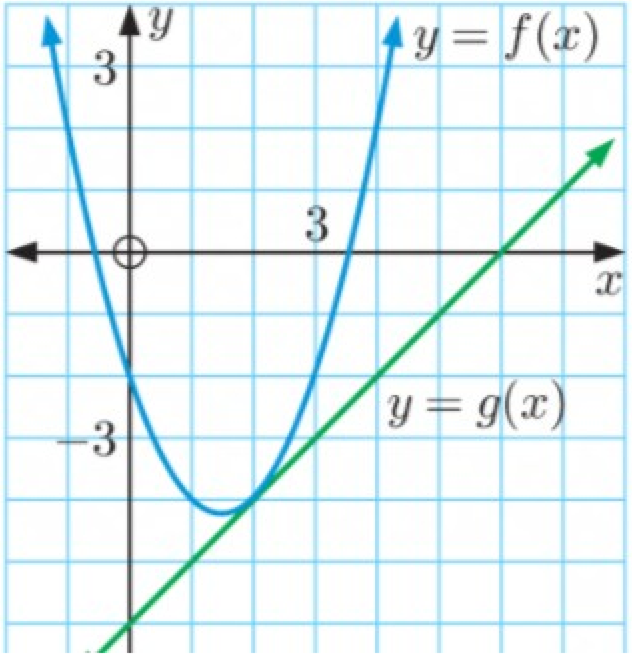


a Find:

i $f(2)$ **ii** $f(3)$

b Find the value of x such that $f(x) = 4$.

8



Consider the graphs of $y = f(x)$ and $y = g(x)$ shown.

a Find:

i $f(4)$ **ii** $g(0)$ **iii** $g(5)$

b Find the *two* values of x such that $f(x) = -2$.

c Find the value of x such that $f(x) = g(x)$.

d Show that $g(x) = x - 6$.

9 Draw a graph of $y = f(x)$ such that $f(-2) = 5$, $f(1) = 0$, and $f(4) = 3$.

10 The graph of $y = f(x)$ is a straight line passing through $(-3, -5)$ and $(1, 7)$.

a Draw the graph of $y = f(x)$.

b Find $f(-3)$ and $f(1)$.

c Find $f(x)$.

THE DOMAIN OF A FUNCTION

To find the domain of a function, we need to consider what values of the variable make the function undefined.

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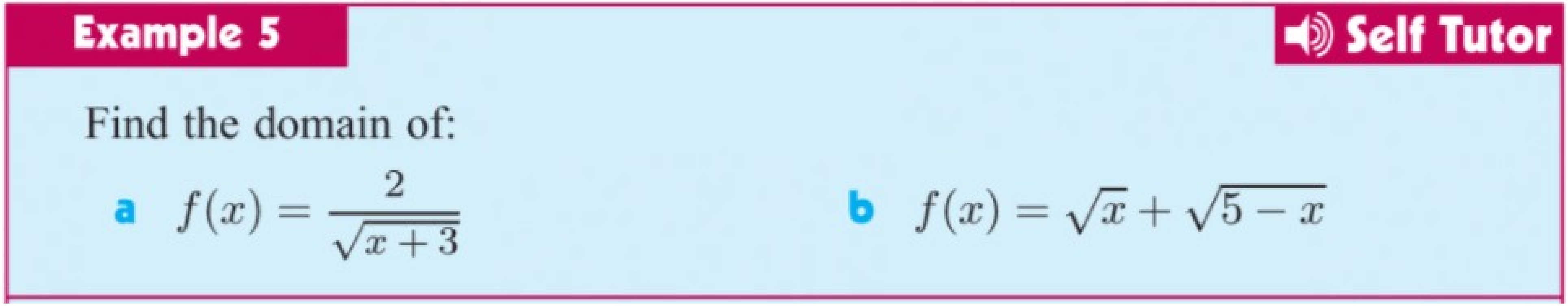
To find the domain of a function, we need to consider what values of the variable make the function undefined.

For example:

- the domain of $f(x) = \sqrt{x}$ is $\{x \mid x \geqslant 0,\ x \in \mathbb{R}\}$, since $\sqrt{x}$ has meaning only when $x \geqslant 0$.
- the domain of $f(x) = \dfrac{1}{\sqrt{x-1}}$ is $\{x \mid x > 1,\ x \in \mathbb{R}\}$ since, when $x - 1 = 0$ we are 'dividing by zero', and when $x - 1 < 0$, $\sqrt{x-1}$ is undefined.

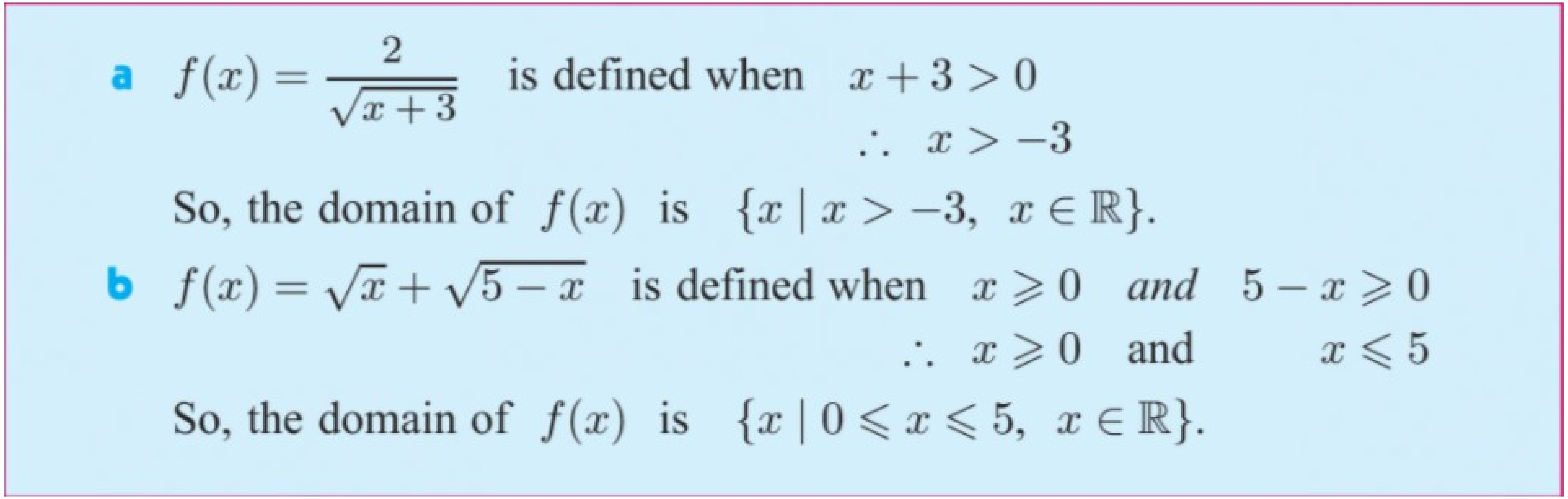


Example 5

Self Tutor

Find the domain of:

a $f(x) = \dfrac{2}{\sqrt{x+3}}$ **b** $f(x) = \sqrt{x} + \sqrt{5-x}$



a $f(x) = \dfrac{2}{\sqrt{x+3}}$ is defined when $x + 3 > 0$

$\therefore \ x > -3$

So, the domain of $f(x)$ is $\{x \mid x > -3, \ x \in \mathbb{R}\}$.

b $f(x) = \sqrt{x} + \sqrt{5-x}$ is defined when $x \geqslant 0$ *and* $5 - x \geqslant 0$

$\therefore \ x \geqslant 0$ and $x \leqslant 5$

So, the domain of $f(x)$ is $\{x \mid 0 \leqslant x \leqslant 5, \ x \in \mathbb{R}\}$.

EXERCISE 15C.2

1 Find the domain of:

a $f(x) = 2x$ **b** $f(x) = \dfrac{1}{x}$ **c** $f(x) = \dfrac{1}{x-3}$

d $f(x) = \dfrac{1}{(x-1)(x+2)}$ **e** $f(x) = \dfrac{x}{x^2-9}$ **f** $f(x) = \dfrac{3}{x^2-5x+4}$

2 Find the domain of:

a $f(x) = \sqrt{x-2}$ **b** $f(x) = \sqrt{3-x}$ **c** $f(x) = \sqrt{x} + \sqrt{2-x}$

d $f(x) = \dfrac{1}{\sqrt{x}}$ **e** $f(x) = \dfrac{1}{\sqrt{x}} + \dfrac{1}{\sqrt{x+2}}$ **f** $f(x) = \dfrac{1}{x\sqrt{4-x}}$

Check your answers using the graphing package.

GRAPHING PACKAGE

EXERCISE 15C.1

1 **a** 3 **b** 7 **c** 1 **d** -7 **e** 2

2 **a** -2 **b** -17 **c** 13 **d** $5x+3$ **e** $-5x-17$

3 **a** 2 **b** 11 **c** 46 **d** $2x^2+3x+2$
e $2x^2+x+1$

4 **a** 45 **b** -3 **c** 0 **d** $4x^2-20x+21$
e $16x^2+8x-3$

5 **a** **i** $-\frac{3}{2}$ **ii** $-\frac{1}{3}$ **iii** $-\frac{8}{3}$ **b** $x=-2$
c $2-\dfrac{7}{x}$ **d** $x=-1$

6 **a** $V(4)=12\,000$. The value of the car after 4 years is \$12 000.
b $V(t)=8000$ when $t=5$. 5 years after purchase the value of the car is \$8000.
c \$28 000

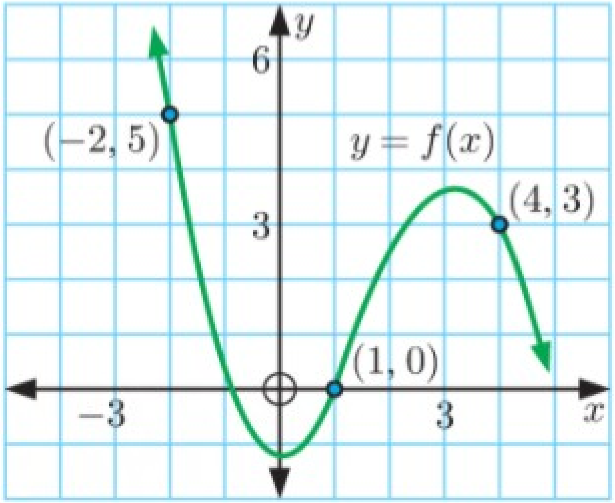
7 **a** **i** $f(2)=1$ **ii** $f(3)=-1$ **b** $x=-4$

8 **a** **i** $f(4)=2$ **ii** $g(0)=-6$ **iii** $g(5)=-1$

b $x = 0$ and $x = 3$ **c** $x = 2$

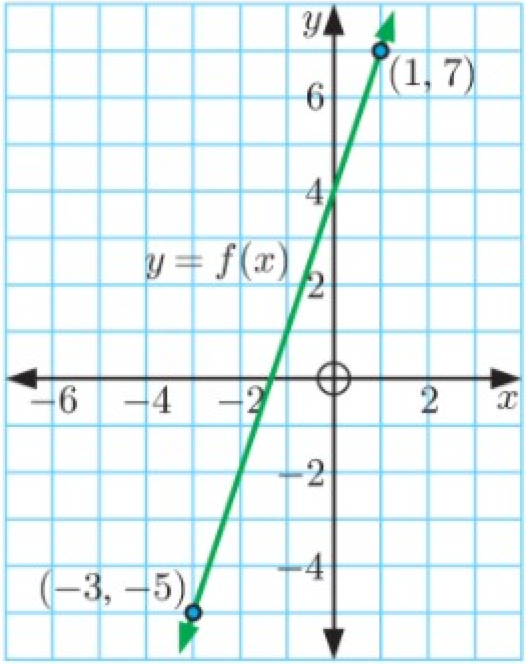
d g has gradient 1 and y-intercept -6.

9



Other graphs are possible.

10 **a**



b $f(-3) = -5$,
$f(1) = 7$

c $f(x) = 3x + 4$

EXERCISE 15C.2

1 a $\{x \mid x \in \mathbb{R}\}$

b $\{x \mid x \neq 0\}$

c $\{x \mid x \neq 3\}$

d $\{x \mid x \neq -2 \text{ and } x \neq 1\}$

e $\{x \mid x \neq 3 \text{ and } x \neq -3\}$

f $\{x \mid x \neq 1 \text{ and } x \neq 4\}$

2 a $\{x \mid x \geqslant 2\}$

b $\{x \mid x \leqslant 3\}$

c $\{x \mid 0 \leqslant x \leqslant 2\}$

d $\{x \mid x > 0\}$

e $\{x \mid x > 0\}$

f $\{x \mid x < 4 \text{ and } x \neq 0\}$